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Bending-active systems: On exploring morphological configurations through coupling with tension-only members

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Abstract

Active deformation applied as primary design agent for the conceptualisation and generation of lightweight free-form structures in architecture, has been recently exponentially adopted by the research community. In principle, elastic members of thin, initially planar geometry of low bending stiffness material, enable the realisation of unique curvilinear structural morphologies, achieved through their capacity in active-bending. Despite this genius concept on producing free-forms, the soft mechanical approach entails great potential to generate more complex systems of enhanced controllability in the active formation process and augmented structural stability and capacity in the post-strained state, through hybridisation with tension-only elements. Along these lines, this paper refers to the investigation of hybrid bending-active and tension-only structural configurations of single and coupled bending-active stripes of regular shape and continuous length. Primary aim of the investigation is to reveal their deformation behavior during the form-finding stage, while preserving their load-bearing capacity in post-formed condition. Activation of the primary elastic members is performed throughout a gradual cable shrinkage process, that also enables the succession of various configuration states of the proposed system. The investigation follows a multi-stage, nonlinear Finite-Element Analysis, considering all nonlinear attributes that govern the members' mechanical and geometrical properties.

Keywords

Hybrid cable bending-active members, Form-finding, Finite-element analysis, Nonlinear analysis, Load-deformation behaviour.

Introduction

The revitalisation of the soft mechanical approach, initially incorporated in the previous Century with the design of strained timber gridshells and plydome structures, is now brought to foreground and re-established within the framework of computational design. Respective current developments run under the influences of new construction techniques, assembly logics and morphological objectives. The realization of these applications is carried out with the support of computational methods and tools provided in software design kits, which enable rapid conception, experimentation and fabrication. Despite the ease workflow and variety of digital design instruments provided by these tools, the design lacks sufficient numerical calculations, which govern the actual nonlinear elastic deformation approach, by excluding necessary information such as nonlinear material characteristics and effects that occur during the active formation process of the elastically deformable elements. In extent, the majority of recent respective applications mainly focuses on stimulating free-form morphologies and organic patterns that are primarily only suitable for small scale constructions, façade skins and building envelopes. In this respect, the design objectives followed leave unexploited structural aspects that active-bending may provide to a system in terms of its low self-weight characteristics and self-stiffening behaviour evoked by the prestress amount acting on the members.

Background

Historically, critical aspects that have triggered the idea of establishing an alternative design and construction approach for the generation of organic forms, followed by active deformation processes of the structural elements, have been mainly based on economic factors (Adriaenssens *et al.*, 2014). Compared to construction frameworks developed for conventional structures, the exploitation of pliable materials of initially straight or planar geometry, overcomes transportation difficulties and minimises construction duration. It suggests the avoidance of cold formed structural elements. In extent, the use of massively produced timber stripes, a relatively cheap and easily accessible material, stretch forth the idea of cost minimisation. Despite related unique and cost-effective construction advantages, design wise, this soft mechanical approach necessitates the use of numerical calculation for the accurate estimation of the active deformation process, in order to efficiently render the actual design outcome. In this respect, the design process superposes linear developments and falls into more complex nonlinear schemes and strategies (Ahlquist *et al.*, 2014).

The aforementioned profound construction and design aspects of the soft mechanical approach can be traced in the example of the Mannheim multihall gridshell, designed by Frei Otto in 1973, and the geodesic plydome project, designed by Buckminster Fuller in 1957 (Liddell *et al.*, 2015). In the first case, the form-finding method adopted for the draft calculation of the structure's deformed shape was investigated following the hanging chain model technique with small scale models. The actual structure is planarly assembled in a regular grid network of cross-fastened rectangular timber stripes and forced to bent using the "pull-up" technique to finally form the synclastic shell shape. The latter example refers to the utilisation of pliable planar plywood plates to generate a self-stabilised double-curvature dome. The overlapping lamination connectivity of the elements, forces the plates to deform in a mutual manner, able to sustain a stabilised three-dimensional dome formation. The hemispheric polyhedron structure preserves global structural rigidity through a self-equilibrium bending force distribution that is acquired from the non-symmetric members' connectivity. The system has been form-found following physical experimentations and intuitive estimation of the elements' overlapping area.

Structural principles and Recent examples

Flexible structures compared to hard mechanical ones, utilize their material elastic capacity in bending, stretching or even buckling to generate transformation. Force is not directly translated into kinetic motion and displacement; instead, it is stored temporarily in the material's molecular structure, i.e. residual force, allowing an incremental, nonlinear form distortion. In particular, bending is not an internal, but rather a visual effect. Within this frame, planar surfaces may form single or double curvature surfaces according to the type and magnitude of the force applied. Transformation takes place so long the force is active. Once the force is released, the elastic members regain their initial form, autonomously and without the need of any additional energy. Such components can also succeed on a vast amount of polymorphic behaviour, due to the highly flexible characteristics of their mechanical attributes (Phocas and Alexandrou, 2017). Lienhard refers to this newly conceptualized idea as 'approach', instead of classifying it as a discrete structural system (Lienhard, 2014).

In engineering terms, the principle structural action, which allows the members' bending-active feature to be suitable for implementation in load-bearing structures, is the pretension force, i.e. embedded energy, stored inside the elastic members, resulting to noticeable structural stiffening of its constituent members (Alexandrou and Phocas, 2016). Henceforth, the active 'rigitization' process can generate solid static solutions of curvilinear configuration with less mass and minimal initial members' three-dimensional geometrical complexity. Consequently, the structures' morphological emergence is directly related with the amount of the prestress acting in the members, the structurally optimised form-found geometry and the desired architectural form. Consequently, there is no absolute architectural freedom in the design approach, but rather a cross referenced, nonlinear design methodology, which should incorporate and validate all aforementioned aspects, before proceeding to the construction phase.

The research pavilion constructed at the University of Stuttgart in 2010 demonstrates how this principle can provide a unique and performative design approach for spatial structures (Leischmann *et al.*, 2012). The overall structure consists of 80 unique birch ply-wood stripes, bent to form a curvilinear highly tensioned torus-like structure, which can attain a valid overall stiffness. At local level, the inversed curvilinear system of the coupling bending components allows the establishment of a force equilibrium state between them. The variation of flexural stresses of all components allows the global structure to reach sufficient robustness, since its stiffness is highly increased, maintaining at the same time the benefits of a lightweight result. The twist installation at the timber expo 2015 in London also reveals how material efficiency can be of significance for the design process (AA, 2015). In this example, low thickness timber profiles are twisted, allowing an increased complexity in the outcome geometry of the deformed structure. The primary forced-to-bend elements exhibit a double-side connection with the secondary planar non-deformed curved elements, in parallel direction. Due to length and planar geometry differentiations, the primary elements are potentially required to both bend and twist, in order to fit the secondary elements planar curvature, enabling thus a set of flexural stresses to be developed. The components' configuration is duplicated along the structure's length, allowing a natural form-finding process to take place, in providing overall stability for the structure.

Assembly and fastening techniques

The ability of an actively-bent structure to succeed and sustain itself in elastically deformed state necessitates the control of its constituent members' boundary conditions or a fully-defined configuration assembly strategy for the members, i.e. members' fastening technique, in case of modular or multi-componential systems. This can be achieved following multiple assembly or load-deformation control techniques and may be classified as follows:

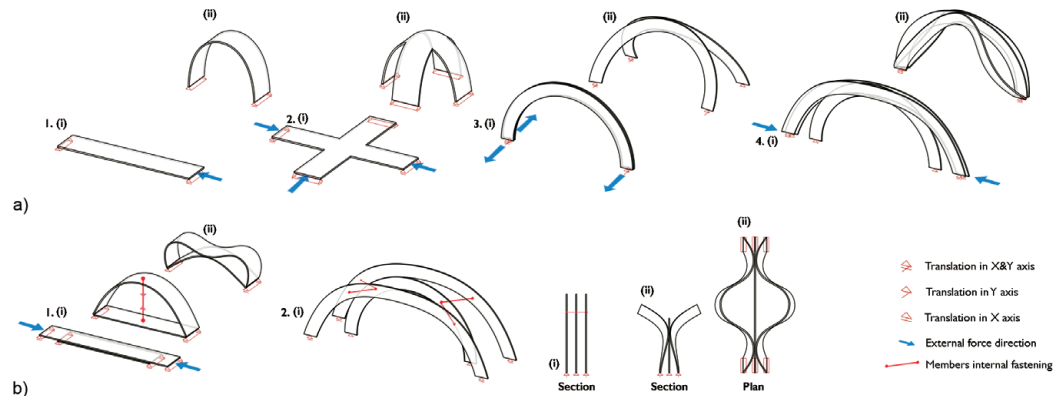
A. Altering boundary conditions. This active formation process refers to planar structural components, composed of single or globally networked elements of pre-fastened plates or stripes, i.e. planar geometries. The process suggests control of the degrees of freedom of the supports to permit motion towards the direction/s that will eventually cause the desired elements' deformation. Practically, the control of the supports' degrees of freedom may be achieved with the design of appropriate structural details, and the activation of motion, through infrastructural support and mechanical actuators. The techniques may vary according to the desired shape acquisition and complexity of the preassembled members (Figure 1a). Applications of such may be found in the construction and erection development of timber gridshells (Liuti, 2016).

B. Coupling or interconnected fastening of elements. This approach suggests a real time partial and stepwise fastening process of elements for the assembly and therefore, the succession of the overall structure's form-found shape. This technique mostly applies to structures that are composed of multiple stripes or plates of similar or dissimilar length. During the interconnected fastening, the system reaches a self-equilibrium state, based on the inner stresses developed in the geometrically indifferent elements employed (Figure 1b). To realize the formation process, an embedded active control system or special mechanical tools are required. This however increases the costs and complexity of construction.

Hybridisation and the role of cable

Due to the assembly and calculation complexity that preoccupies the soft mechanical approach, bending-active systems necessitate consideration of both, structural behaviour and formative construction aspects, to be encompassed in the final construction, in order to reach a valid and performative lightweight outcome. Along these lines, the current paper suggests the embedment of secondary cable elements, to function both as actuation means for the active formation process, as well as stabilizers and structural strengthening add-ons in the hybrid structure's post from-found state. This synergic structural action between bending-active and tension-only members may also provide the advantage of reaching and sustaining a broader spectrum of multiple configurational states, followed by the active regulation of the cables' length.

In hybrid tensile and tensegrity structural systems, which are frequently applied in architecture for achieving large spans, the role of the cables is very critical and aims at providing adequate geometrical stiffness to the structure (Saitoh and Okada, 1999). This is controlled and further optimized through introduction of pretension. The dome example presented in (Quagliaroli *et al.*, 2015), clearly demonstrates the superiority of self-stress stiffening achieved by using calibrated cable pretensioning, compared to the mechanical rigidity provided by a bracing technique, for both load-deformation control and self-weight minimisation.

**Figure 1.**

- a) Altering boundary conditions: Examples of simple elastic structural configurations. The active formation process is enabled through control of translational freedom of the supports; (i) non-deformed state; (ii) deformed state
- b) Examples of the coupling or interconnected fastening technique of group of elastic members. Case I is developed from an initially horizontal configuration and case two from a vertical; (i) non-deformed state; (ii) deformed state

In hybrid cable bending-active systems, the role of cables falls into more complex set of criteria with regard to the evaluation of their actual role in the design and structural efficiency. Due to the active deformation of the primary elastic members, the cables become significant components for the overall shape acquisition of the system. This is primarily based on the need of the elastic members to rest in static equilibrium in their deformed state. Subsequently, the role of cables may be characterised as integral. Compared to hybrid rigid elements, in which the cables can be utilised for either passive or active applications, the relation that governs hybrid flexible systems is rather singular, that of active control, applied through an initial cable's length reduction. This action simultaneously induces prestress in both elements, i.e. the cables and the elastic members, and results to global stiffening effects (Figure 2). In this respect, the following aspects may characterise the general contribution of cables in bending-active systems:

1. As actuators for erection and form-finding of the hybrid system, through their initial length reduction.
2. As control elements for stiffening and inducing prestress to the hybrid system.

Typological Investigation

The typological investigation focuses on the morphological exploration emerging from single curvature configurations of the members' bending in one direction only. The hybrid structural prototypes are composed of single and coupled regular bending-active stripes of uniform rectangular section, which are connected with a single cable with variable length. All units are based on possible derivatives and more complex geometrical configurations of the primal, most fundamental configuration of a two-element unification, unit I, as shown in Figure 3. Unit I consists of a 1.0 m long stripe with sectional dimensions of 250 x 10 mm (width to thickness), connected on both ends with a single cable element of 10 mm diameter. The stripes are being assigned to PTFE material with an

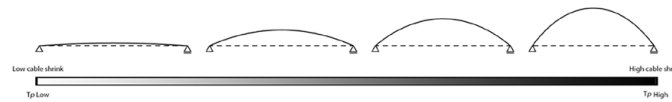


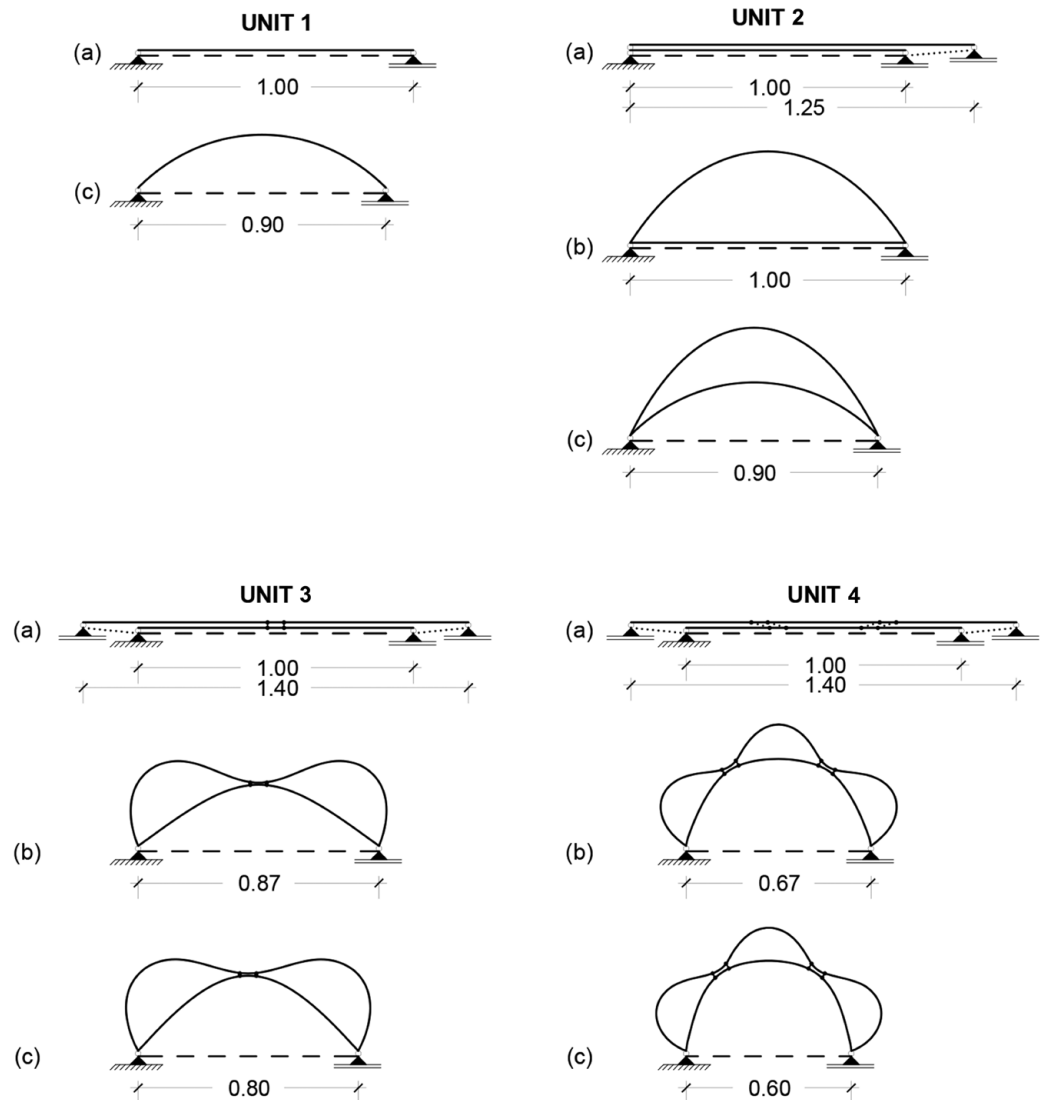
Figure 2.

Simple hybrid cable bending-active configuration. Cable shrinkage length as actuation mean for controlling the bending-active member's curvature. Gradual cable shrinkage results to proportional axial forces increase

elastic modulus of 2.5 GPa and are ground supported on both ends with one support allowing rolling in longitudinal direction, X-axis, and both supports allowing rotation along the Y-axis. Unit 2 is composed of two bending-active members of 1.0 and 1.4 m respectively. The deformed shape results into a double arch deformed configuration. In unit 3 and 4, single and double interconnected stripe fastening is applied to the members at midspan, $L/2$ and $L/3$ of the actual length respectively. The fastening of the members is simulated using secondary contracting cables of shrinkable length.

Numerical Analysis

For improved accuracy, the deformation and stress analysis of the units is conducted with the Finite-Element Analysis (FEA) software, SOFiSTiK® (SOFiSTiK AG, 2014). In contrast to computational tools and calculation methods (Veenendaal and Block, 2012) provided by alternative software, FEA considers nonlinear effects that take place during the active-deformation, such as large deflections of the system and length modifications, and uses the member's mechanical characteristics, which necessitate the calculations. All units have been geometrically defined with the modelling software Rhino® (Robert McNeel & Associates, 2017) and then exported in SOFiSTiK. For the establishment of a sequential nonlinear analysis workflow, with the ability to provide custom settings in each analysis step, the Teddy module (alternative text input platform) provided by SOFiSTiK has been selected (SOFiSTiK AG: Basics, 2014). Teddy allows customisation of the analysis sequence and every new load-case to act additionally on the previously stored ones. In this respect, all internal forces developed from previous load-cases in the structural members are been stored and considered as starting conditions for the following one (SOFiSTiK AG: ASE, 2014). The general workflow followed is demonstrated in Figure 4.


Figure 3.

Proposed cable bending-active configurations; (a) Non-deformed state; (b) Bending-active members fastened state; (c) Form-found state following main cable's length reduction (Phocas and Alexandrou, 2016)

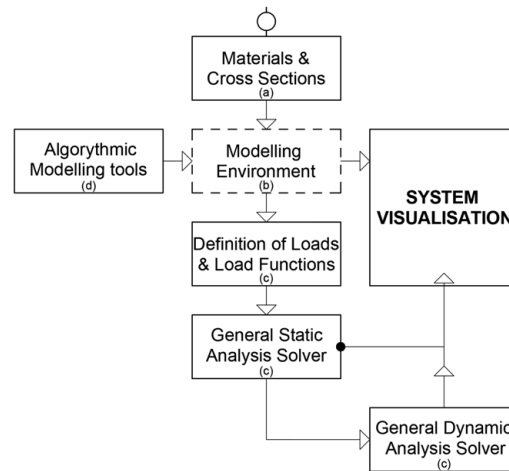


Figure 4.

FEA workflow; (a) Performed in SOFiStiK main interface; (b) and (d) enabled in third party modelling environment; (c) enabled via text commands in Teddy

Analysis Stages

The analysis model seeks to create sequential nonlinear analysis stages that follow in analogues way the hypothesised actual physical structure's assembly. Subsequently, the model generates the form-found shapes of the units. All four units are designed in planar shape and bent in progression sequences to reach half their initial span length ($L_0/2$). The target span has been purposely selected for the analysis to provide insight in the elastic deformation capacity of the units. The analysis process is analytically described in (Phocas and Alexandrou, 2016). Excluding unit 1, which only necessitated a single analysis stage, that of its main cable's length reduction, all other units have been form-found following two stages. In the first stage, the two components have been fastened together using secondary cables of shrinkable lengths and in the second stage, the units have been further deformed through their main cable's length reduction. The secondary cable elements used in stage 1 are necessarily employed for simulation means, in order to enable the fastening process of the units. In actual construction, this assembly method may be performed using mechanical actuators or specialised mechanical equipment. In stage 2, the units have been further deformed through the main cable's length reduction, until the desired deformed span length is reached at half of the initial non-deformed length $L_0/2$. As a result, a set of multiple analyses are needed to take place in sequential way, until the overall structures' shape is achieved. The form-found shapes of the units in all analysis stages are demonstrated in Figure 5.

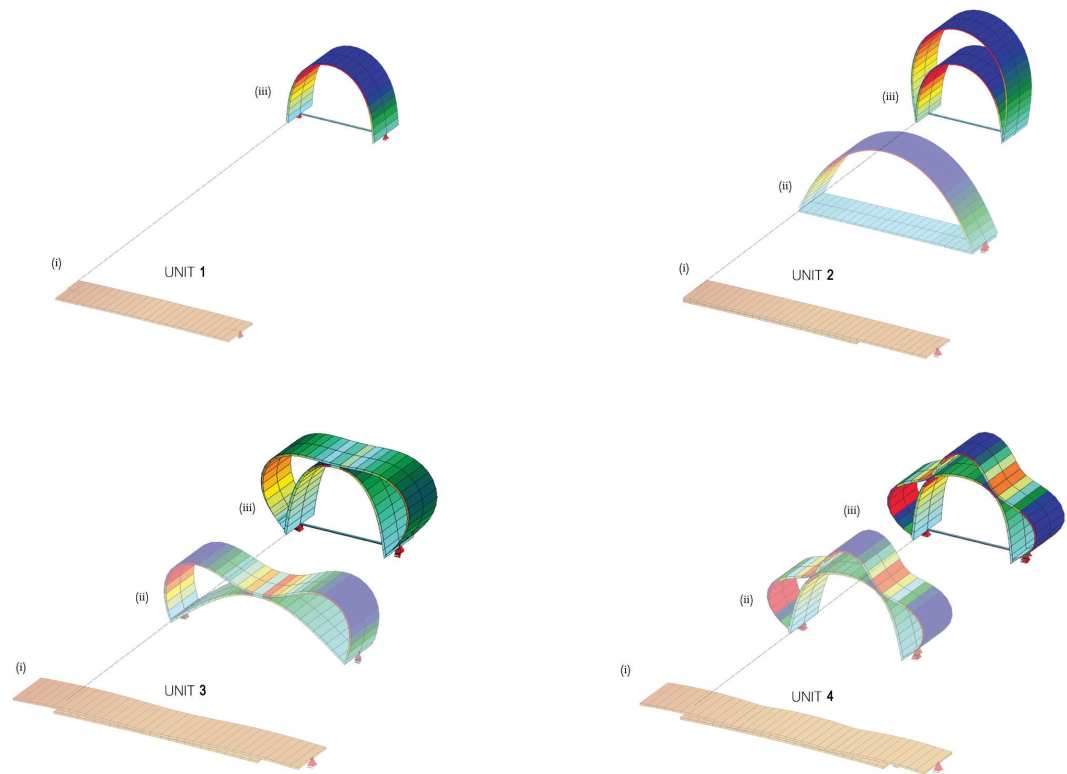


Figure 5.

Unit's form-found shapes, obtained through FEA; (i) Non-deformed shape; (ii) Internally fastened bending-active members form-found shape; (iii) Form-found shape after main cable's length reduction (Phocas and Alexandrou, 2016)

Morphological Variations

Due to the elastic members' inter-fastening process performed in unit 3 and 4 and implemented in stage I, the bending-active members' length differentiation causes a mutual influence on their geometry resulting to distinguished global curvature configurations. This is directly related to the initial distance between the two members that undergo fastening. In principle, the fastening process enables the units to substantially deform upward and reduce their span. Therefore, a relatively lower cable shrinkage value is needed for these units in stage 2 to reach the desired deformed span of $L_0/2$.

The coupling technique responsible for the units' active deformation control, as examined in unit 3 and 4, may enable the desirable shape succession and can be considered as ideal form generating design aspect. However, the elastic members' coupling provides only limited geometrical adaptability to the system. In contrast, this option can be extensively handled on the basis of the hybridisation of the system, i.e. by the cable element, which can act both, as actuator and as a strengthening member in improving the load-bearing capacity of the structure. For this reason, in following the paper focuses on further analysis of unit 1, in examining its active configurability possibilities.

System Development

Unit I has been examined with longer system span of 4.0 m and interconnected at five points using four cables of 1.0 m initial length. Each cable acts autonomously in terms of its length reduction, in order to investigate how this affects the primary member's deformed shape, and therefore trace the geometrical limitations of the hybrid system configuration. However, for creating a comparable set of results, a preliminary uniform cable length reduction has been performed in a preliminary stage, i.e. form-found system, to potentially deform the structure at $L_0/2$ (i.e. 2.0 m). This value has been purposely selected as it allows the system to reach an appropriate span to height ratio of 0.675 (Table 1). Once the first stage has been completed, the initially sliding ground support is fixed. Throughout this process 12 morphological differentiated shapes for all the scenarios of discretised cables action have been obtained. Due to symmetric geometrical characteristics of the system, all mirrored cases have been excluded from the results and the resulting six unique cable activation scenarios are demonstrated in Figure 6.

Numerical Results

The deformed shape of the form-found structure, as obtained from the preliminary analysis in stage I, reaches an absolute maximum stress of the bending-active member of approximately 40 % (18.80 MPa) of the material's bending strength of 48.00 MPa. The resulting configuration refers to a uniform cable's length reduction of 66.2 mm. The cables' initial prestress causes an average axial tension of 0.46 kN. For the activation case scenarios, the system is examined in its maximum deformation, obtained by gradually increasing the amount of the cable's shrinkage value to the point that the bending-active member reaches an absolute stress value, slightly smaller than the maximum bending strength of the material used (PTFE). The active cables are denoted from left to right with C1...C4. The analysis results of all examined scenarios are included in Table 2 and Figure 7. The maximum cable's axial force developed reaches an absolute value of 1.45 kN in case I.4. In all cases, the cable's maximum axial force development is not related to the cable that performed activation. In case I.1 and I.5, maximum values of the cable's shrinkage are achieved with values of 260 and 220 mm respectively. In these cases, a higher amount of cable shrinkage may be achieved, because the acting cables link the bending-active member with the ground support. In this respect, the cable's length modification causes the global structure to rotate, instead of inducing local deformation on the bending-active member. This can also be detected from the resulting span to height ratio of the structure, corresponding to relative low values of 0.545 and 0.493 respectively. A minimum cable's shrinkage value of only 140 mm is observed in case I.6, resulting into a higher symmetrically deformed structure shape of 0.666 span to height ratio.

Conclusions

The current paper refers to the design, analysis and re-configurability control of four hybrid cable bending-active units. The preliminary investigation of the prototype units with 1.0 m span employs both, single and coupled bending-active stripe configurations followed by the main cable's length reduction to examine their reconfiguration potentials in terms of their geometrically deformed capacity. It has been observed that the geometrical deformation already caused in stage I of the coupled cases, limits the reconfiguration capacity of the units to be further controlled by the cable element in consecutive stage. Therefore, the subsequent investigation scheme focused on a longer bending-active stripe with 4.0 m span, interconnected with multiple cables at five points. The results obtained clearly demonstrate the morphological reconfiguration capacity that may be achieved in six custom cable activation scenarios. The proposed design and construction techniques presented in the current paper demonstrate the hybridisation potentials emerging from the two-element unification in achieving a highly reconfigurable system, while maintaining self-stabilisation and low self-weight structural assembly. In extent, the integral role of the cable to act both, as a system's erection instrument and post-deformation control component, renders a simplified approach in dealing with flexible structures.

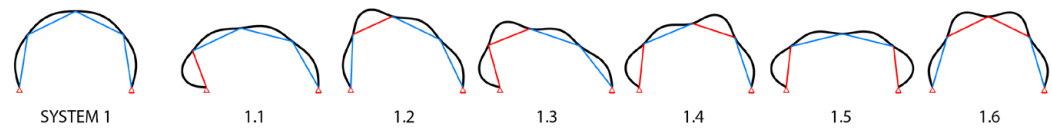


Figure 6.

System I deformation limits for all six cable activation cases

Table 1.

System I reconfigurations. Maximum stresses of the bending-active members and span to height system ratio

System Configurations	Max. Stress [MPa]	Span/height Ratio
Form-Found System	18.80	0.675
System I.1	48.00	0.545
System I.2	47.65	0.683
System I.3	47.53	0.580
System I.4	47.85	0.644
System I.5	47.11	0.493
System I.6	47.21	0.666

Table 2.

System I reconfigurations. Maximum cable's axial force and activation cable's shrinkage value

	Cable Elements			
	C1	C2	C3	C4
Form-Found System				
Cable's Axial Force [kN]	0.48	0.44	0.44	0.48
Cable's Shrinkage [mm]	66.2	66.2	66.2	66.2
System I.1				
Cable's Axial Force [kN]	0.62	0.82	0.85	0.66
Cable's Shrinkage [mm]	260	-	-	-
System I.2				
Cable's Axial Force [kN]	0.81	1.04	1.26	0.81
Cable's Shrinkage [mm]	-	190	-	-
System I.3				
Cable's Axial Force [kN]	0.72	1.15	1.45	0.93
Cable's Shrinkage [mm]	170	170	-	-
System I.4				
Cable's Axial Force [kN]	0.75	1.43	1.16	0.93
Cable's Shrinkage [mm]	170	-	170	-
System I.5				
Cable's Axial Force [kN]	0.72	1.21	1.21	0.72
Cable's Shrinkage [mm]	220	-	-	220
System I.6				
Cable's Axial Force [kN]	0.98	1.28	1.23	0.98
Cable's Shrinkage [mm]	-	140	140	-

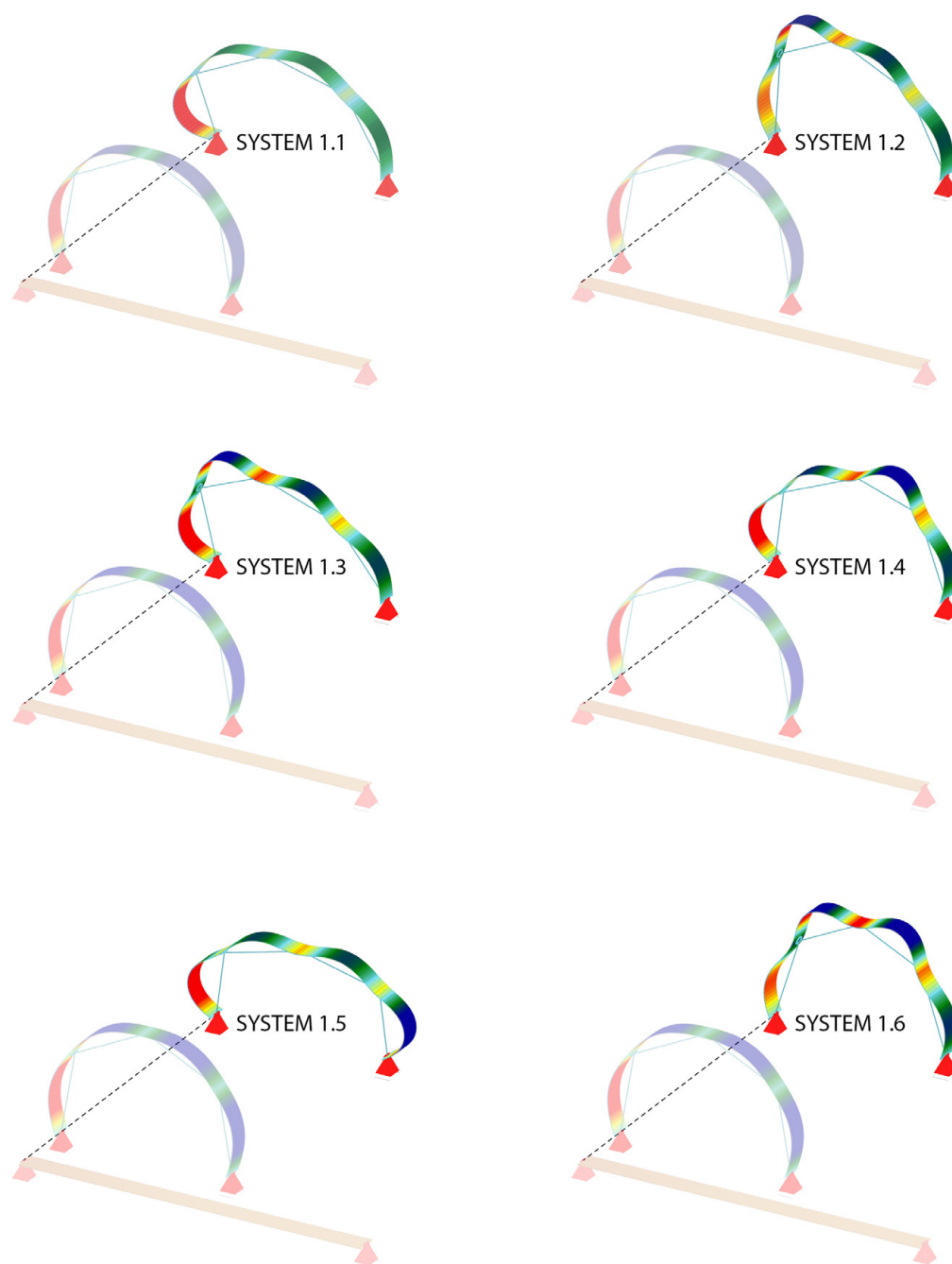


Figure 7.

System 1 reconfigurations. Non-deformed and deformed system configurations using specific cables activation

In architecture, compared to conventional design schemes followed by the employment of rigid members and realised on the basis of a top-down approach, the soft mechanical approach renders the possibility of generating organic shapes through repetitive nonlinear geometrical form-finding processes. Despite its engineering novelty in dealing with natural shapes, the soft approach ends up becoming complex and time consuming in cases where complex shapes are desired. This is directly affected by several parameters, such as the members' planar shape and construction details with regard to the fastening technique applied for the elastic members. The approach presented in the current paper may enable a simplified design approach towards generating curvilinear structural forms. The use of regular planar stripe members and the introduction of tension-only elements enable a respective morphological exploration process of bending active members. The internally connected cable elements provide segmental deformation to the elastic members enabling in this respect higher amounts of deformation control at global scale. Further research will focus in large scale structures with increased number of elastic members and cable elements, in order to extend the typological investigation of this approach. The construction and design simplicity of the concept makes it a promising solution for lightweight reconfigurable structures.

References

AA Twist installation 2015: Webpage: <http://www.archdaily.com/775842/emtechs-twist-displayed-at-the-timber-expo-in-birmingham> Accessed 21 Nov. 2016.

Adriaenssens, S., Block, P., Veenendaal, D. and Williams, C., 2014. *Shell Structures for Architecture*. Routledge. doi: 10.4324/9781315849270.

Ahlquist, S., Kampowski, T., Torghabehi, O. O., Menges, A. and Speck, T., 2014. Development of a digital framework for the computation of complex material and morphological behavior of biological and technological systems. *Computer-Aided Design*, 60, pp. 84–104. doi: 10.1016/j.cad.2014.01.013.

Alexandrou, K. and Phocas, M.C., 2016. Bending-active structures :A parametric analysis on decoding structural behavior and capacity. Cruz, P.J.S. (ed.) *Structures and Architecture: Concepts, Application and Challenges, Third International Conference on Structures & Architecture, ICSA 2016, 27.07 – 29.07.16*, Guimaraes, Portugal, London: Taylor & Francis, pp. 1061–1068.

Leischmann, M., Knippers, J., Lienhard, J., Menges, A. and Schleicher, S., 2012. Material behaviour. *Architectural Design*, 216, pp. 44–51.

Liddell, I., 2015. Frei Otto and the development of gridshells. *Case Studies in Structural Engineering*, 4, pp. 39–49. doi: 10.1016/j.csse.2015.08.001.

Lienhard, J., 2014. Bending-Active Structures: form-finding strategies using elastic deformation in static and kinematic systems and the structural potentials therein. University of Stuttgart: Institute of Building Structures and Structural Design.

Liuti, A., 2016. Erection of post-formed gridshells by means of inflatable membrane technology membrane technology. In Stephan, R. H. C. and A. (ed.) *International Conference of the Architectural Science Association 2015*. Melbourne: Taylor & Francis, pp. 678–687.

Phocas, M.C. and Alexandrou, K., 2016. On decoding the structural behaviour of hybrid cable bending-active units in fastening, prestress and load-bearing state. *Space Structures*, (under review).

Phocas, M.C. and Alexandrou, K., 2017. Adaptive Structures. Soft Mechanical Approach. *Special Issue: Mobile, Adaptable and Rapidly Assembled Structures, Computational Methods and Experimental Measurements*, 5(4), pp. 421–431.

Quagliaroli, M., Malerba, P.G., Albertin, A. and Pollini, N., 2015. The role of prestress and its optimization in cable domes design. *Computers and Structures*, 161, pp. 17–30. doi: 10.1016/j.compstruc.2015.08.017.

Robert McNeel & Associates, 2017. Rhinoceros 5.0 Service Release, computer program, Seattle, North America: ROBERT MCNEEL & ASSOCIATES.

Saitoh, M. and Okada, A., 1999. The role of string in hybrid string structure. *Engineering Structures*, 21(8), pp. 756–769. doi: 10.1016/S0141-0296(98)00029-7.

Schleicher, S., Rastetter, A., La Magna, R., Schönbrunner, A., Haber Bosch, N. and Knippers, J., 2015. Form-Finding and Design Potentials of Bending-Active Plate Structures. Edited by F. S. Mette Ramsgaard Thomsen, M. Tamke, C. Gengnagel and B. Faircloth, *Modelling Behaviour: Design Modelling Symposium 2015*. Zurich: Springer, pp. 53–63. doi: 10.1007/978-3-319-24208-8.

SOFiSTiK AG., 2014. ASE: General Static Analysis of Finite Element Structures. Oberschleissheim, Germany: SOFiSTiK AG.

SOFiSTiK AG., 2014. SOFiSTiK: Basics. Version 2016-6. Oberschleissheim, Germany: SOFiSTiK AG, pp. 51–58.

Veenendaal, D. and Block, P., 2012. An overview and comparison of structural form finding methods for general networks. *Solids and Structures*, 49(26), pp. 3741–3753. doi: 10.1016/j.ijsolstr.2012.08.008.